

AN APPROXIMATE METHOD OF CALCULATING THE HEAT TRANSFER COEFFICIENT IN METAL BARS

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A least-squares method is used to determine the heat transfer coefficient H of a solid copper bar in air at constant temperature. For this purpose, twelve steady states and their corresponding cooling curves were measured with temperature excesses of the metal over the surrounding air in the range of 11–74°C and were compared to those calculated using a mathematical model to solve the equation for the heat flow in the bar. The model reduces an experimental double exponential law to an overall single exponential and gives similar behavior, within 10% of statistical uncertainty, for H in the steady state and in cooling. The values obtained in this study are in qualitative agreement with the values given in the literature under similar experimental conditions, but there it is not specified how they are obtained in solids.

1. Introduction. Unlike the thermal conductivity K , general methods are lacking to determine the heat transfer coefficient H accurately. This is because H depends on many factors: the type of flow (laminar or turbulent), the mechanism of the heat transfer (forced or natural convection), the nature and geometry of the body, etc. Only for flow over bodies having a simple geometry, such as a flat plate, or flow inside a circular tube can the overall H be calculated more or less accurately (Dittus–Boelter equation) in terms of the Nusselt number [1]. But for flow over bodies of complex configuration or solids of different nature and shape the task of evaluating the coefficient H becomes too complicated and an experimental approach must be used to determine it. One knows *a priori* that a wide range of values of H may be found for different applications. A comparison, for example, of film condensation theory with experiments has shown that the measured heat transfer coefficient is about 20% higher than the theory suggests, and consequently appropriate corrections must be taken into account in the theory [2].

However, in recent years, numerical methods and computer simulation have become the usual tools that enable one to approach many of the problems related to heat transfer and the thermal coefficients involved. Some of the most significant and recent examples are Ko and Bert's method of calculating the exact solution of the two-dimensional steady-state conduction problem of a rectangular region with convective boundary conditions in both directions [3], Kamiuto's two-parameter formula for predicting the total effective thermal conductivities of alumina-silica fiber insulations with an accuracy of $\pm 30\%$ [4], Spitzer's least-squares method used to determine the heat flux from solidifying steel into a water-cooled steel belt [5], Viswanath et al.'s general method for obtaining polynomials from experimental data [6], Nance et al.'s work that describes parallel application of direct simulation (the Monte Carlo method) to three-dimensional flow over a flat plate [7], etc.

In this paper we will try to give a more general method to obtain the coefficient H for solids by measurements of several steady states and their corresponding cooling by using a least-squares method to find the best governing equations for the experimental data. As we will show, even for the very simple geometric ar-

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rangement of a metallic copper rod, calculation of the heat transfer coefficient presents practical and theoretical difficulties. One of the reasons resides in the nature of the heat-flow differential equation itself, which has more complicated solutions than one would have expected a priori. Section 2 briefly describes the theoretical model and its solution for one-dimensional heat flow in a solid. Section 3 sets up our generalized solution by fitting the experimental data with the best governing equation describing the temperature distribution in a specific solid, and also describes the methodology and the experimental procedure. Section 4 gives results for the steady state and cooling, leading to the discussion and conclusions in Sec. 5.

2. Mathematical Model. The mathematical model to predict the behavior of the heat transfer coefficient in a solid is not as easy to study as one might expect. In principle, it is based on Newton's cooling law, which states that a solid at a higher temperature than the surroundings transfers heat across its surface at a rate proportional to the temperature difference between the solid and the medium. This constant cooling ratio is H . Inclusion of this concept in the differential equation for the heat flow in a thin bar gives [8]

$$\frac{\partial T}{\partial t} = \frac{K}{\rho c} \frac{\partial^2 T}{\partial x^2} - \frac{Hp}{\rho cw} (T - T_0). \quad (1)$$

The coefficient H can be obtained on the basis of Eq. (1) either from the steady-state solution, $(\partial T/\partial t) = 0$, as

$$\frac{d^2 T_i}{dx^2} - m^2 T_i = 0, \quad T_i = T_1 \exp(-mx), \quad (2)$$

where $m^2 = Hp/Kw$, or from the cooling solution, $(\partial^2 T/\partial x^2) = 0$, as

$$\frac{dT'}{dt} + m' T' = 0, \quad T' = T_i \exp(-m't), \quad (3)$$

where $m' = Hp/\rho cw$.

Here T is the temperature excess over the surroundings, T_1 is the highest temperature in the bar, and T' is the instantaneous temperature at time t during the cooling at point i of the bar, which started from the steady state at temperature T_i .

If we consider now the reasonable hypothesis that the heat transfer coefficient is independent of the kind of regime established in the solid, i.e., whether steady state or cooling, it is possible to eliminate H from the definitions of m and m' in Eqs. (2) and (3). One obtains the theoretical relation between the exponents

$$m' = \frac{K}{\rho c} m^2, \quad (4)$$

which should be very easy to check by study of the steady state and the corresponding cooling curves. Thus, whether Eq. (4) is verified or not depends on the independence or dependence of H on the geometric and/or thermodynamic variables. A problem arises when the actual experimental conditions start to differ from Newton's conditions [8]: if they are close, one expects a constant-like behavior for H , but if they are far apart, one expects that the coefficient H will no longer be constant and an appropriate functional equation ought to be sought.

The fact that experimentally the steady state and cooling actually follow exponential laws is an immediate consequence of H being constant, since if H depends on any other factor such as t , x , and/or T , then extra terms must be added to Eq. (1). The second- and first-order differential equations (Eqs. (2) and (3), respectively) are then no longer valid, and neither are their respective exponential solutions for H . In other words, one could not predict an a priori behavior for the coefficient H because of the strong dependence on these variables. Instead it is necessary to ascertain, in so far as it is possible, the behavior of H for cases that are much easier to study. However, previous results have shown that even cases close to the simplest conditions

lead to different conclusions: while Eq. (4) is verified (within 1.2% of statistical uncertainty) in an iron bar for a temperature range of 11-61°C over the surroundings [9], it is not verified either for an aluminium bar over a range of 20-74°C [10] or for a transparent plastic (polymethyl methacrylate) bar over the same range of temperatures [11] (both with a statistical uncertainty of 32%). Taking into account these contrasting results, it was therefore decided to study the range of validity of Eq. (4) for good conducting materials, such as copper, in the hope that a theory predicting the value of the heat transfer coefficient could be developed on the basis of finding the best governing equation during the steady state and cooling and then relating the parameters of the fit to thermal coefficients such as K and H .

3. The Solution Proposed and the Experimental Procedure. Since an earlier experiment [12] had shown that the single exponential solutions given by Eqs. (2) and (3) were not good at describing the heat transfer coefficient, it was decided to search for the best governing equations by fitting the experimental data (in the steady state and cooling) to pre-chosen functions and calculating the coefficients by a least-squares method [13, 14] using the STEPIT statistical package, which uses matrix inversion and automatic step size adjustment [15]. These functions must be chosen from a range of possible solutions. Taking into account the nature of the material, previous results [9-12], and the usual methodology in this kind of study [5, 6], the expressions chosen to fit the experimental data were double exponents for the steady state and cooling:

$$T_i = a_1 \exp(-m_1 x) + a_2 \exp(-m_2 x), \quad (5)$$

$$T' = a'_1 \exp(-m'_1 t) + a'_2 \exp(-m'_2 t), \quad (6)$$

respectively.

Since the experimental temperature regime used was not very much above the ambient temperature, one can still follow the simple model of heat flow described by Eq. (1) with its solutions being single exponents. If the purpose is to identify the double exponential equation that was obtained as the best governing equation for the experimental data with the theoretical single exponential solution described by Eqs. (2) and (3), one must reduce the double exponent to a single exponent in some way. Then, if we consider that Eqs. (2) and (3) can be satisfied and the double exponential law is the best governing equation for the steady state and cooling, it should be a reasonable hypothesis to consider that an overall exponential coefficient can be obtained for the steady state (m in Eq. (2)) and cooling (m' in Eq. (3)) in terms of the combined partial coefficients m_1 and m_2 in Eq. (5), and m'_1 and m'_2 in Eq. (6), thus:

$$m = -\frac{1}{x} \ln \left[\frac{1}{T_1} (a_1 \exp(-m_1 x) + a_2 \exp(-m_2 x)) \right], \quad (7)$$

$$m' = -\frac{1}{t} \ln \left[\frac{1}{T_i} (a'_1 \exp(-m'_1 t) + a'_2 \exp(-m'_2 t)) \right]. \quad (8)$$

The overall coefficient H can then be obtained from Eqs. (2) and (3) and their comparison will be a direct check of satisfiability of Eq. (4). Thus, Eqs. (7) and (8) are the equations that represent the model we propose for obtaining the heat transfer coefficient.

Table 1 lists the physical properties [16] and geometric characteristics (with their experimental errors) of the copper bar. The bar was held in two wooden supports, isolated by asbestos cord, and heated electrically at one end by a resistive coil connected to an ac variable transformer. The experimental data were obtained from thermocouples (with an accuracy of $\pm 0.1^\circ\text{C}$) placed in ten holes drilled perpendicular into the bar from a generatrix to the axis, and the temperatures were recorded automatically over a sufficiently long period of time. The first thermocouple was at 7 cm from the heated end and was taken as the origin for the distances. Table 2 lists the distances (with an uncertainty of ± 0.1 cm) of the rest of the thermocouples with respect to the first, and Table 3 lists the temperatures in the first thermocouple for every steady state studied, which are also the

TABLE 1. Physical and Geometric Characteristics of the Copper Bar at 20°C with Their Experimental Errors

$$\begin{aligned} \rho &= (8.933 \pm 0.001) \cdot 10^3 \text{ kg} \cdot \text{m}^{-3} \text{ (at } 25^\circ\text{C)} \\ c &= (3.87 \pm 0.01) \cdot 10^2 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1} \text{ (at } 25^\circ\text{C)} \\ K &= (3.83 \pm 0.01) \cdot 10^2 \text{ J} \cdot \text{sec}^{-1} \cdot \text{m}^{-1} \cdot \text{K}^{-1} \text{ (at } 50^\circ\text{C)} \\ k &= (1.108 \pm 0.006) \cdot 10^{-4} \text{ m}^2 \cdot \text{sec}^{-1} \\ L &= 0.420 \pm 0.001 \text{ m} \\ d &= (2.500 \pm 0.005) \cdot 10^{-2} \text{ m} \\ p &= (7.85 \pm 0.02) \cdot 10^{-2} \text{ m} \\ w &= (4.91 \pm 0.02) \cdot 10^{-4} \text{ m}^2 \end{aligned}$$

TABLE 2. Distance of the Thermocouples with Respect to the First, Which Is Taken as the Coordinate Origin

Points	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Distance (± 0.1 cm)	0.0	3.6	7.1	10.6	14.0	17.5	20.9	24.4	27.9	31.4

TABLE 3. The Starting Experimental Conditions T_1 , the Temperature at $x_1 = 0$, for the Twelve Steady States and Coolings Studied

Cases studied	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
$T_1 (\pm 0.2^\circ\text{C})$	77.4	70.2	66.9	57.6	51.1	44.2	40.4	34.8	30.8	24.1	20.4	14.2

starting temperatures for the corresponding cooling cases. Twelve steady-state cases were studied with an overall range of temperature excess over the temperature of the surroundings (63.2°C) from 77.4°C in case I to 14.2°C in case XII. These steady-state cases and their corresponding cooling behaviors were then used to obtain the best governing equations and hopefully shed some light on the behavior of the heat transfer coefficient.

4. Results. Prior to further calculations one must know which kind of differential equation for the heat flow is to be used. First, since the cylinder has a small cross section compared to its length (see Table 1), the problem is one of linear flow in which the temperature is specified by the time and the distance x measured along the rod, as given by Eq. (1). Second, and much more importantly, Eq. (3) is satisfied only when a lumped-system analysis is assumed, i.e., when the Biot number $Bi = HL/K$ is less than 0.1. If $Bi > 0.1$, then Eq. (3) is no longer valid and the finite-difference representation of the time-dependent heat conduction equation must be applied to the experimental data [17]. As we did not know a priori the behavior of H , these finite-difference equations were applied to our experimental data. The results, however, were not consistent with those expected for non-lumped systems. The lumped-system analysis was therefore assumed and Eq. (3) was considered to be valid for our study.

4.1. Steady state. Table 4 shows results of fitting the steady-state experimental data to a double exponential law as in Eq. (5). The standard deviation is denoted by σ and the last column, *sqdif*, is the sum of the squares of the differences between the experimental data T_{ie} and the theoretical values T_{it} , i.e. $\sum_i^N (T_{ie} - T_{it})^2$.

This quantity is a measure of the goodness of the fit. The results are both clear and interesting: clear because the *sqdif* values are so small that one can indeed conclude that the best governing equation for the steady state in the copper bar is Eq. (5) (comparing the experimental values for the temperature of the first thermocouple at $x = 0$ (Table 3) with the theoretical values $(a_1 + a_2)$ in Table 4, one can see how good the fit is); interesting, because of the importance of the physical meaning of the results, since there were three classes of behavior. Cases I–III and X show an m_2 value that is so small that the second exponent is reduced to the constant a_2 . This was the behavior found for an aluminium bar in all the cases studied [10]. Case XII shows two equal exponents, so that the double exponential expression reduces to a single exponent with $(a_1 + a_2)$ very close to the value of T_1 of Table 3. This was the behavior found for an iron bar, also for all the cases studied [9]. The

TABLE 4. Parameters for the Fit of the Experimental Steady-State Data to the Double Exponential Law of Eq. (5), Units of a_1 and a_2 Are ($^{\circ}\text{C}$) and Units of m_1 and m_2 Are (m^{-1})

Cases	$a_1 \pm \sigma$	$m_1 \pm \sigma$	$a_2 \pm \sigma$	$m_2 \pm \sigma$	$sqdif$
I	24.39 ± 0.04	5.74 ± 0.04	53.1 ± 0.6	$< 10^{-6}$	0.26
II	22.36 ± 0.08	5.82 ± 0.05	47.8 ± 0.3	"	0.11
III	421.28 ± 0.04	5.754 ± 0.04	45.7 ± 0.6	"	0.14
IV	19.80 ± 0.06	6.10 ± 0.05	41.7 ± 0.4	0.09 ± 0.03	0.14
V	14.42 ± 0.06	6.17 ± 0.05	36.8 ± 0.4	0.09 ± 0.03	0.15
VI	11.04 ± 0.05	6.61 ± 0.05	33.2 ± 0.4	0.14 ± 0.03	0.09
VII	11.40 ± 0.05	5.15 ± 0.05	29.2 ± 0.4	0.06 ± 0.03	0.17
VIII	8.61 ± 0.05	6.62 ± 0.05	26.2 ± 0.3	0.15 ± 0.03	0.04
IX	8.91 ± 0.05	4.96 ± 0.04	22.0 ± 0.3	0.04 ± 0.03	0.10
X	7.00 ± 0.04	5.65 ± 0.04	17.2 ± 0.3	$< 10^{-6}$	0.04
XI	4.79 ± 0.04	5.44 ± 0.04	15.7 ± 0.3	0.15 ± 0.03	0.05
XII	5.74 ± 0.04	0.84 ± 0.04	8.2 ± 0.4	0.85 ± 0.04	0.33

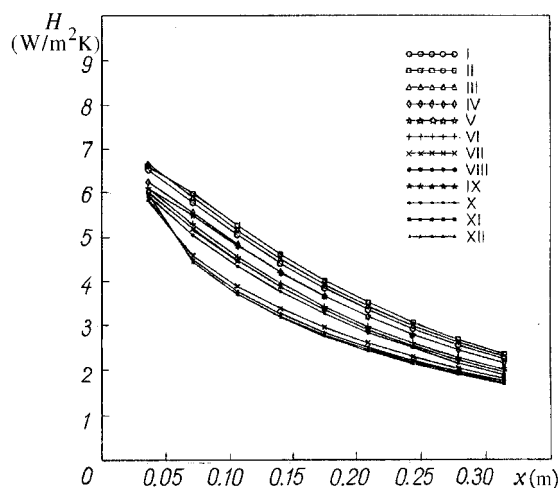


Fig. 1. Heat transfer coefficient in the steady state for the cases studied.

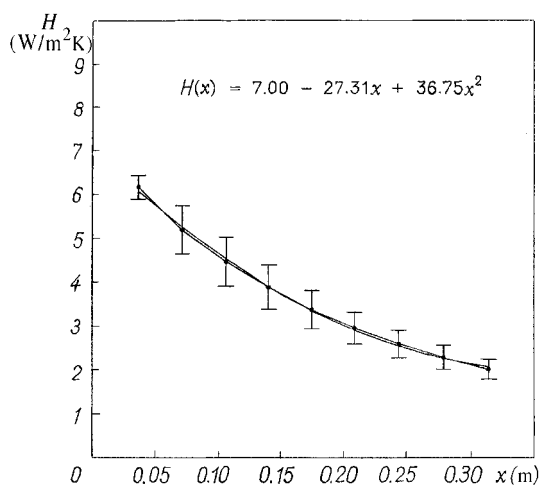


Fig. 2. The same as Fig. 1, but taking the mean value at each point, with its standard error and the best fit to a second-order polynomial.

rest the cases, IV–IX and XI, present a second exponent that is small relative to the first exponent. This is a behavior that had not been found in previous steady-state experiments, even though it is well-known in cooling [9, 12].

The calculated coefficients in Table 4 were used in Eq. (7) to obtain an overall m value to calculate H from Eq. (2). Figure 1 plots those H values vs x . It is of interest to note that the curves are almost interwoven, not obeying the common rule "upper lines, upper temperatures," even though there is a natural tendency toward lower curves at lower temperatures. The curves corresponding to cases IX, XI and XII are completely indistinguishable. Figure 2 shows the mean values for the twelve cases studied at each point of the bar and their standard deviation. The final H curve obeys a second-order polynomial in x , as shown in the figure, with a very high accuracy of $sqdif = 0.026$.

4.2. *Cooling.* Once the steady states were reached, cooling was initiated by switching off the electric heater at the end of the bar. There are twelve different coolings at the ten points of the bar. Since this involves a great quantity of experimental data, three points of the bar were chosen for better comprehension of the results: $x_1 = 0$, $x_2 = 0.036$ m, and $x_5 = 0.14$ m. All the cooling measurements started from the experimental

TABLE 5. Values of the Fit of the Experimental Data to the Double Exponential Law of Eq. (6) for the Cooling of the First Point of the Bar, Units of a'_1 and a'_2 Are ($^{\circ}\text{C}$), and Units of m'_1 and m'_2 Are (sec^{-1})

Cases	$a'_1 \pm \sigma$	$(m'_1 \pm \sigma) \cdot 10^{-3}$	$a'_2 \pm \sigma$	$(m'_2 \pm \sigma) \cdot 10^{-3}$	<i>sqdif</i>
I	50.0 ± 1.0	0.30 ± 0.008	32.895 ± 0.003	0.583 ± 0.003	0.43
II	46.2 ± 0.9	0.335 ± 0.009	26.984 ± 0.004	0.800 ± 0.004	0.18
III	48.1 ± 0.9	0.342 ± 0.008	22.422 ± 0.004	0.883 ± 0.004	0.09
IV	39.2 ± 0.9	0.33 ± 0.01	20.422 ± 0.005	0.824 ± 0.005	0.16
V	38.2 ± 0.9	0.34 ± 0.01	15.194 ± 0.005	0.899 ± 0.005	0.18
VI	30.4 ± 1.1	0.30 ± 0.01	16.846 ± 0.005	0.553 ± 0.005	0.16
VII	30.0 ± 1.1	0.31 ± 0.01	15.142 ± 0.006	0.554 ± 0.005	0.59
VIII	28.6 ± 0.9	0.34 ± 0.01	7.582 ± 0.007	1.019 ± 0.007	0.05
IX	17.9 ± 3.0	0.26 ± 0.06	16.134 ± 0.008	0.534 ± 0.008	0.20
X	17.9 ± 1.0	0.32 ± 0.02	7.75 ± 0.01	0.80 ± 0.01	0.03
XI	13.7 ± 4.4	0.35 ± 0.2	8.38 ± 0.02	0.36 ± 0.02	0.10
XII	1.7 ± 1.8	2.2 ± 0.2	13.3 ± 0.2	0.37 ± 0.02	0.03

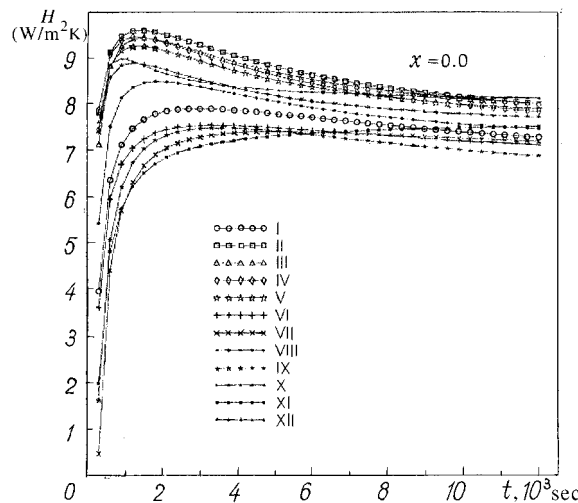


Fig. 3. Heat transfer coefficient during cooling for the cases studied at $x = 0.0$ m.

values of T_1 listed in Table 3 and the fits were performed with the same mathematical expressions as for the steady state, i.e., Eqs. (6) and (8) in this case.

Data were logged every 300 sec up to 12,000 sec (with an experimental error for the measurements of 1 sec), depending on the case, until the temperature had dropped to 1.0 - 1.7°C over the temperature of the surroundings. As an example, Table 5 lists the results of the fit to Eq. (6) for the double exponential law for the first hole at $x_1 = 0$, where the temperature excess was highest. Two clear classes of behavior can be seen separating cases I–X and XI, XII. The exponents in cases I–X are qualitatively the same, but quantitatively different: both can be considered as fluctuating around their mean values, but while m'_1 is smaller and mainly constant with small fluctuations, m'_2 , on average, is about two and a half times greater than the corresponding m'_1 , and its fluctuations are considerably smaller. Cases XI, XII behave quite differently because both reduce to a single exponent, albeit in different ways. Case XI has the same exponents with different coefficients whose sum gives the same value as the single exponent. By contrast, case XII has two quite different exponents: the first is about six times greater than the second. This means a very sharp decay of the first exponent with respect to the second, and since the first coefficient a'_1 is much smaller than the second coefficient a'_2 , one can

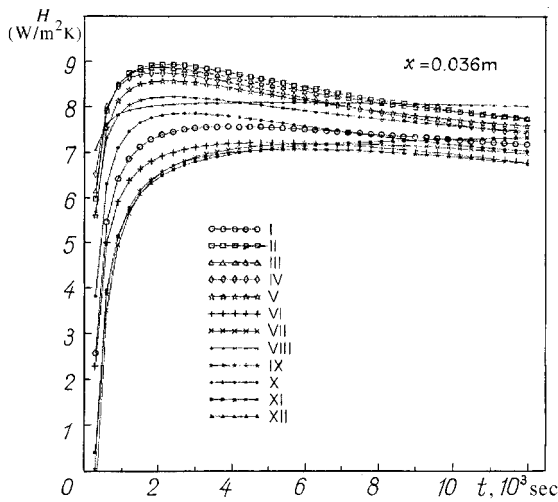


Fig. 4. The same as Fig. 3, but for $x = 0.036$ m.

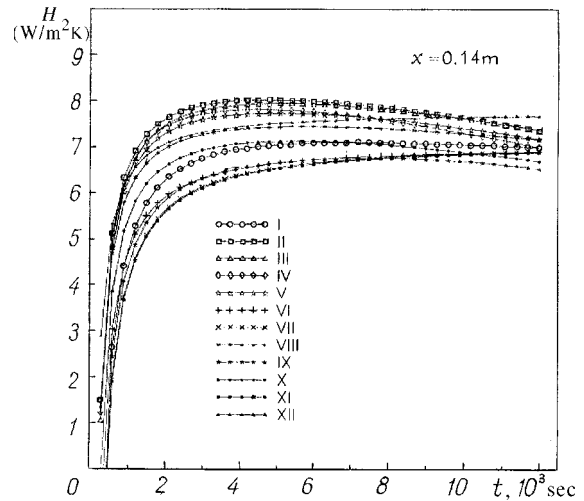


Fig. 5. The same as Fig. 3, but for $x = 0.14$ m.

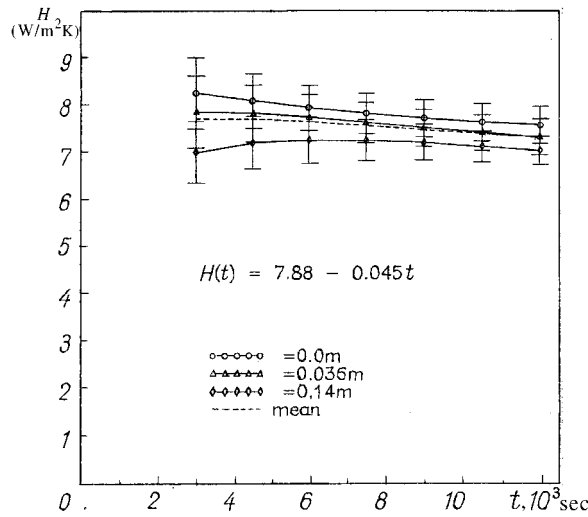


Fig. 6. The same as Figs. 3-5, but taking the mean value and its standard deviation, and avoiding the unsteady cooling. The dashed line is the mean of the three values at each time.

conclude that the first exponent can be considered as a "perturbation" of the second exponent, which truly governs the time decay of the temperature.

Figure 3 shows the time behavior of H at $x = 0$, when the values of m'_1 and m'_2 of Table 5 were taken to obtain the overall m' of Eq. (8) and then Eq. (3). Figures 4 and 5 correspond to the same procedure, but for $x = 0.036$ and $x = 0.14$, where the corresponding tables for the coefficients m'_1 and m'_2 have been omitted for brevity. As one can see from the figures, three important things are found. First, as expected from the theory [18], in all the cases there exists unsteady cooling, associated with the thermal inertia of the heater and the bar, from the beginning of the cooling and lasting about $1/6$ of the total cooling time, some 2000 sec, being sharper for the farthest point in the bar and smoother for the inner points. Second, the curves interweave, as they did for the steady state, indicating a tendency toward a common behavior. Third, and more importantly, all the values tend to almost the same constant value, even though there is a weak tendency toward lower values at inner points in the bar. This can be seen better in Fig. 6, where the mean values (and their standard deviation) of all the coolings at several points along the bar have been plotted, avoiding, as usual, the first part corresponding to the unsteady cooling regime. The dotted line is the mean value for the three cases. It is almost constant, since the slope of the best fit, given in the figure, is very close to zero with $sqdif = 0.005$.

5. Discussion and Conclusions. We used a least-squares method [15] to fit the experimental results from the steady state and cooling to the best governing equation for the temperature distribution in these two regimes. The pre-chosen analytical functions were exponents and polynomials of n -th-order. However, the least-squares method shows that the double exponential expressions (Eqs. (5) and (6)) can be considered to be the best fit for the steady state and cooling, respectively, in the copper bar, since the alternative fit to an n -th-order polynomial (not seen here for simplicity) was, for all the cases studied, very poor. The results obtained with a fourth-order polynomial were unsatisfactory for the steady states and were worse for the cooling, with a *sqdif* value ranging between 2.0 and 3.5 for cases I-V. Only a sixth-order polynomial gave an acceptable fit. However, it is still worse than was obtained for the double exponential law given in Table 5, especially for cases I-V.

Associated with this application of regression to our experimental data, there arises a very important and interesting situation: the way in which the experimental double exponential law can be related to the theoretical single exponential law given by Eqs. (2) and (3). This led us to propose our approximate solution in which we consider the theoretical single exponent as a generalized linear combination of two different exponents given by Eqs. (7) and (8), where the coefficients are obtained directly from the experimental data, Tables 4 and 5.

Applying this model to the heat transfer coefficient calculations, one can draw the following conclusions from our results, summarized in Figs. 2 and 6:

a) The Newton cooling law assumes a constant value for H , independent of the temperature and time. However, in the range of temperatures studied here for the steady state and cooling, we found two different behaviors for the two regimes: while for the cooling regime one can consider an almost constant value for H , for the steady-state regime H was found to decay with distance according to a second-order polynomial.

b) Comparing the heat transfer coefficients obtained from the steady state and cooling, one can see that in the limiting case for the initial conditions ($x = 0$ and $t = 0$) these two values of H are the same within a 10% uncertainty, as can be noted in Figs. 2 and 6, in which $H(x = 0) = 7.00$ and $H(t = 0) = 7.88$. The same ratio is found for the overall exponents in Eq. (4).

c) The results obtained in this study improve considerably results for similar previous experiments on iron, aluminium, and plastic [9, 12], since in the present study the model described by Eqs. (7) and (8) was used. The values found here for H are in qualitative agreement with experiments with similar experimental conditions where the coefficient H is not calculated but is taken as given and it is not stated how it is obtained [17].

Finally, this work is one of the very few devoted to obtaining the qualitative behavior of the heat transfer coefficient for a solid, since it is striking in the literature on this topic that there is a scarcity of methods for obtaining H experimentally or analytically, even though there appear from time to time notes [19] or comments [20] about the peculiarities and difficulties in determining this coefficient.

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NOTATION

c , specific heat; H , heat transfer coefficient; K , thermal conductivity; p , perimeter; t , time; T , temperature; T_0 , constant temperature of the medium; w , cross section; x , coordinate in the axial direction; *sqdif*, sum of the squares of the differences between the experimental data and the theoretical values; $\kappa = K/\rho c$, thermal diffusivity; ρ , density; σ , standard deviation.

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